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$B \rightarrow K^{(*)} \bar{K}^{(*)}$ decays: a new method for measuring the CP phase α

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Abstract

We present a new method for measuring the CP phase α . It requires the measurement of the pure penguin decays $B_d^0(t) \rightarrow K^{(*)} \bar{K}^{(*)}$ and $B_s^0 \rightarrow K^{(*)} \bar{K}^{(*)}$. The method is quite clean: we estimate the theoretical uncertainty to be at most 5%. By applying the method to several $K^{(*)} \bar{K}^{(*)}$ final states, α can be extracted with a fourfold ambiguity. An additional assumption reduces this ambiguity to twofold: $\{\alpha, \alpha + \pi\}$. Since no π^0 detection is needed, this method can be used at hadron colliders.

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There is a great deal of excitement these days regarding CP violation in the B system. The latest measurements of the CP-violating phase β have now produced definitive evidence for CP violation outside the kaon system [1]:

$$\sin 2\beta = 0.79 \pm 0.12. \quad (1)$$

The ultimate goal of the study of CP-violating rate asymmetries in B decays is to measure each of the interior angles of the unitarity triangle [2], α , β and γ . In this way we will be able to test the standard model (SM) explanation of CP violation. B -factories have obtained β by measuring the CP asymmetry in the “gold-plated” decay mode $B_d^0(t) \rightarrow J/\psi K_S$. And many methods have been proposed for measuring, or putting limits on, the CP phase γ [2].

On the other hand, to date there are only two clean techniques for the extraction of α , and each has its particular difficulties. In the first method, one uses the CP asymmetry in $B_d^0(t) \rightarrow \pi^+ \pi^-$ to obtain α . However, in order to remove the penguin “pollution”, it is necessary to perform an isospin analysis of $B \rightarrow \pi\pi$ decays [3], which includes the measurement of $B_d^0 \rightarrow \pi^0 \pi^0$. Since the branching ratio for this decay is expected to be quite small, it may be very difficult to obtain α in this way. Second, one can use a Dalitz-plot analysis of $B_d^0(t) \rightarrow \rho\pi \rightarrow \pi^+ \pi^- \pi^0$ decays [4]. The problem here is that one must understand the continuum background to such decays with considerable accuracy, as well as the correct description of $\rho \rightarrow \pi\pi$ decays, and again these may be difficult. Note also that both methods require the detection of π^0 ’s, which makes them a challenge for hadron colliders.

In this Letter, we present a new method for measuring α based on the pure penguin decays $B_d^0(t) \rightarrow$

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$K^{(*)}\bar{K}^{(*)}$ and $B_s^0 \rightarrow K^{(*)}\bar{K}^{(*)}$, which are related by U -spin. By studying these decays in the limit of heavy-quark symmetry, chiral symmetry, and the large-energy limit of QCD for the final-state kaons, we argue that the $SU(3)$ -breaking effects are quite a bit smaller than what is usually found. In particular, our best estimate of the theoretical error in our method is at most 5%, which makes the extraction of α quite clean. Note that it is possible to make a variety of independent experimental measurements which will test the claim of small $SU(3)$ breaking.

Because the branching ratios for $B_d^0(t) \rightarrow K^{(*)}\bar{K}^{(*)}$ are rather small, and because B_s^0 decays are involved, this method is probably most appropriate for hadron colliders, particularly since no π^0 detection is needed. Still, it is not out of the question that e^+e^- B -factories might be able to use this technique. One potential drawback of this method is the presence of multiple discrete ambiguities. However, by combining information from several final $K^{(*)}\bar{K}^{(*)}$ states, it is possible to reduce the ambiguity in α to a fourfold one. And by imposing a further (reasonable) theoretical condition, one can obtain only a twofold ambiguity: $\{\alpha, \alpha + \pi\}$.

Consider the pure $b \rightarrow d$ penguin decay $B_d^0 \rightarrow K^0\bar{K}^0$. At the quark level, the decay takes the form $\bar{b} \rightarrow \bar{d}s\bar{s}$. The amplitude can be written

$$\begin{aligned} A(B_d^0 \rightarrow K^0\bar{K}^0) &= P_u V_{ub}^* V_{ud} + P_c V_{cb}^* V_{cd} + P_t V_{tb}^* V_{td} \\ &= \mathcal{P}_{uc} e^{i\gamma} e^{i\delta_{uc}} + \mathcal{P}_{tc} e^{-i\beta} e^{i\delta_{tc}}, \end{aligned} \quad (2)$$

where $\mathcal{P}_{uc} \equiv |(P_u - P_c) V_{ub}^* V_{ud}|$, $\mathcal{P}_{tc} \equiv |(P_t - P_c) \times V_{tb}^* V_{td}|$, and we have explicitly written out the strong phases δ_{uc} and δ_{tc} , as well as the weak phases β and γ . (In passing from the first line to the second, we have used the unitarity of the Cabibbo–Kobayashi–Maskawa (CKM) matrix, $V_{ub}^* V_{ud} + V_{cb}^* V_{cd} + V_{tb}^* V_{td} = 0$, to eliminate the $V_{cb}^* V_{cd}$ term.) The amplitude \bar{A} describing the conjugate decay $\bar{B}_d^0 \rightarrow K^0\bar{K}^0$ can be obtained from the above by changing the signs of the weak phases.

By making time-dependent measurements of $B_d^0(t) \rightarrow K^0\bar{K}^0$, one can obtain the three observables

$$\begin{aligned} X &\equiv \frac{1}{2}(|A|^2 + |\bar{A}|^2) \\ &= \mathcal{P}_{uc}^2 + \mathcal{P}_{tc}^2 - 2\mathcal{P}_{uc}\mathcal{P}_{tc} \cos \Delta \cos \alpha, \end{aligned}$$

$$Y \equiv \frac{1}{2}(|A|^2 - |\bar{A}|^2) = -2\mathcal{P}_{uc}\mathcal{P}_{tc} \sin \Delta \sin \alpha,$$

$$\begin{aligned} Z_I &\equiv \text{Im}(e^{-2i\beta} A^* \bar{A}) \\ &= \mathcal{P}_{uc}^2 \sin 2\alpha - 2\mathcal{P}_{uc}\mathcal{P}_{tc} \cos \Delta \sin \alpha, \end{aligned} \quad (3)$$

where $\Delta \equiv \delta_{uc} - \delta_{tc}$. It is useful to define a fourth observable:

$$\begin{aligned} Z_R &\equiv \text{Re}(e^{-2i\beta} A^* \bar{A}) \\ &= \mathcal{P}_{uc}^2 \cos 2\alpha + \mathcal{P}_{tc}^2 - 2\mathcal{P}_{uc}\mathcal{P}_{tc} \cos \Delta \cos \alpha. \end{aligned} \quad (4)$$

The quantity Z_R is not independent of the other three observables:

$$Z_R^2 = X^2 - Y^2 - Z_I^2. \quad (5)$$

Thus, one can obtain Z_R from measurements of X , Y and Z_I , up to a sign ambiguity. Note that the three independent observables depend on four theoretical parameters (\mathcal{P}_{uc} , \mathcal{P}_{tc} , Δ , α), so that one cannot obtain CP phase information from these measurements [5]. However, one can partially solve the equations to obtain

$$\mathcal{P}_{tc}^2 = \frac{Z_R \cos 2\alpha + Z_I \sin 2\alpha - X}{\cos 2\alpha - 1}. \quad (6)$$

Now consider a second pure $b \rightarrow d$ penguin decay of the form $B_d^0 \rightarrow K^* \bar{K}^*$. Here K^* represents any excited neutral kaon, such as $K^*(892)$, $K_1(1270)$, etc. This second decay can be treated completely analogously to the first one above, with unprimed parameters and observables being replaced by primed ones. One can then combine measurements of the two decays to obtain

$$\frac{\mathcal{P}_{tc}^2}{\mathcal{P}_{tc}'^2} = \frac{Z_I \sin 2\alpha + Z_R \cos 2\alpha - X}{Z_I' \sin 2\alpha + Z_R' \cos 2\alpha - X'}. \quad (7)$$

Now comes the main point. The ratio $\mathcal{P}_{tc}^2/\mathcal{P}_{tc}'^2$ can be obtained by measuring B_s^0 decays to the same final states $K^0\bar{K}^0$ and $K^*\bar{K}^*$. Consider first the decay $B_s^0 \rightarrow K^0\bar{K}^0$. This is described by a $b \rightarrow s$ penguin amplitude:

$$\begin{aligned} A(B_s^0 \rightarrow K^0\bar{K}^0) &= P_u^{(s)} V_{ub}^* V_{us} + P_c^{(s)} V_{cb}^* V_{cs} + P_t^{(s)} V_{tb}^* V_{ts} \\ &\simeq (P_t^{(s)} - P_c^{(s)}) V_{tb}^* V_{ts} \equiv \mathcal{P}_{tc}^{(s)}. \end{aligned} \quad (8)$$

In writing the second line, we have again used the unitarity of the CKM matrix to eliminate the $V_{cb}^* V_{cd}$

piece. Furthermore, the $V_{ub}^* V_{us}$ piece is negligible: $|V_{ub}^* V_{us}| \ll |V_{tb}^* V_{ts}|$. Thus, the measurement of the branching ratio for $B_s^0 \rightarrow K^0 \bar{K}^0$ yields $|\mathcal{P}_{tc}^{(s)}|$. Similarly one can obtain $|\mathcal{P}_{tc}'^{(s)}|$ from the branching ratio for $B_s^0 \rightarrow K^* \bar{K}^*$. However, to a very good approximation,

$$\frac{\mathcal{P}_{tc}^{(s)2}}{\mathcal{P}_{tc}'^{(s)2}} = \frac{\mathcal{P}_{tc}^2}{\mathcal{P}_{tc}'^2}. \quad (9)$$

(Note that the CKM matrix elements cancel in both ratios.) As we will argue below, the theoretical error in making this approximation is at most 5%. The measurements of the branching ratios for $B_s^0 \rightarrow K^0 \bar{K}^0$ and $B_s^0 \rightarrow K^* \bar{K}^*$ will therefore allow one to obtain $\mathcal{P}_{tc}^2/\mathcal{P}_{tc}'^2$. Thus, by combining Eqs. (7) and (9), one can extract α quite cleanly (up to discrete ambiguities, which will be discussed below).

A modification of this method can also be used when the final state is not self-conjugate. For example, consider the decay $B_d^0 \rightarrow K^0 \bar{K}^*$. As for the above processes, the amplitude can be written

$$A(B_d^0 \rightarrow K^0 \bar{K}^*) = \tilde{\mathcal{P}}_{uc} e^{i\gamma} e^{i\tilde{\delta}_{uc}} + \tilde{\mathcal{P}}_{tc} e^{-i\beta} e^{i\tilde{\delta}_{tc}}. \quad (10)$$

(The hadronic parameters are written with tildes to distinguish them from their counterparts in Eq. (2).) For this decay, the amplitude \bar{A} for $\bar{B}_d^0 \rightarrow K^0 \bar{K}^*$ is not simply related to that for $B_d^0 \rightarrow K^0 \bar{K}^*$ since the hadronization is different: in the latter decay, the spectator quark is part of the K^0 , while in the former it is contained in the \bar{K}^* . We therefore write

$$A(\bar{B}_d^0 \rightarrow K^0 \bar{K}^*) = \tilde{\mathcal{P}}_{uc}' e^{-i\gamma} e^{i\tilde{\delta}_{uc}'} + \tilde{\mathcal{P}}_{tc}' e^{i\beta} e^{i\tilde{\delta}_{tc}'}. \quad (11)$$

By measuring $B_d^0(t) \rightarrow K^0 \bar{K}^*$, one can obtain the observables X, Y, Z_I, Z_R defined previously. These now take the form

$$\begin{aligned} X &= \frac{1}{2} [\tilde{\mathcal{P}}_{uc}^2 + \tilde{\mathcal{P}}_{tc}^2 - 2\tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc} \cos(\alpha - \tilde{\Delta}) \\ &\quad + \tilde{\mathcal{P}}_{uc}'^2 + \tilde{\mathcal{P}}_{tc}'^2 - 2\tilde{\mathcal{P}}_{uc}'\tilde{\mathcal{P}}_{tc}' \cos(\alpha + \tilde{\Delta}')], \\ Y &= \frac{1}{2} [\tilde{\mathcal{P}}_{uc}^2 + \tilde{\mathcal{P}}_{tc}^2 - 2\tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc} \cos(\alpha - \tilde{\Delta}) \\ &\quad - \tilde{\mathcal{P}}_{uc}'^2 - \tilde{\mathcal{P}}_{tc}'^2 + 2\tilde{\mathcal{P}}_{uc}'\tilde{\mathcal{P}}_{tc}' \cos(\alpha + \tilde{\Delta}')], \\ Z_I &= \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{uc}' \sin(2\alpha - \tilde{\Delta} + \tilde{\Delta}') \\ &\quad - \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc}' \sin(\alpha - \tilde{\Delta}) - \tilde{\mathcal{P}}_{tc}\tilde{\mathcal{P}}_{uc}' \sin(\alpha + \tilde{\Delta}'), \end{aligned}$$

$$\begin{aligned} Z_R &= \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{uc}' \cos(2\alpha - \tilde{\Delta} + \tilde{\Delta}') + \tilde{\mathcal{P}}_{tc}\tilde{\mathcal{P}}_{tc}' \\ &\quad - \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc}' \cos(\alpha - \tilde{\Delta}) - \tilde{\mathcal{P}}_{tc}\tilde{\mathcal{P}}_{uc}' \cos(\alpha + \tilde{\Delta}'), \end{aligned} \quad (12)$$

where $\tilde{\Delta} \equiv \tilde{\delta}_{uc} - \tilde{\delta}_{tc}$ and $\tilde{\Delta}' \equiv \tilde{\delta}_{uc}' - \tilde{\delta}_{tc}'$.

For the second process, it is natural to consider the conjugate final state $\bar{K}^0 K^*$. The amplitudes for B_d^0 and \bar{B}_d^0 to decay to this state are

$$\begin{aligned} A(B_d^0 \rightarrow \bar{K}^0 K^*) &= \tilde{\mathcal{P}}_{uc}' e^{i\gamma} e^{i\tilde{\delta}_{uc}'} + \tilde{\mathcal{P}}_{tc}' e^{-i\beta} e^{i\tilde{\delta}_{tc}'}, \\ A(\bar{B}_d^0 \rightarrow \bar{K}^0 K^*) &= \tilde{\mathcal{P}}_{uc} e^{-i\gamma} e^{i\tilde{\delta}_{uc}} + \tilde{\mathcal{P}}_{tc} e^{i\beta} e^{i\tilde{\delta}_{tc}}. \end{aligned} \quad (13)$$

Measurements of $B_d^0(t) \rightarrow \bar{K}^0 K^*$ then yield

$$\begin{aligned} X' &= \frac{1}{2} [\tilde{\mathcal{P}}_{uc}'^2 + \tilde{\mathcal{P}}_{tc}'^2 - 2\tilde{\mathcal{P}}_{uc}'\tilde{\mathcal{P}}_{tc}' \cos(\alpha - \tilde{\Delta}') \\ &\quad + \tilde{\mathcal{P}}_{uc}^2 + \tilde{\mathcal{P}}_{tc}^2 - 2\tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc} \cos(\alpha + \tilde{\Delta})], \\ Y' &= \frac{1}{2} [\tilde{\mathcal{P}}_{uc}'^2 + \tilde{\mathcal{P}}_{tc}'^2 - 2\tilde{\mathcal{P}}_{uc}'\tilde{\mathcal{P}}_{tc}' \cos(\alpha - \tilde{\Delta}') \\ &\quad - \tilde{\mathcal{P}}_{uc}^2 - \tilde{\mathcal{P}}_{tc}^2 + 2\tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc} \cos(\alpha + \tilde{\Delta})], \\ Z_I' &= \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{uc}' \sin(2\alpha + \tilde{\Delta} - \tilde{\Delta}') \\ &\quad - \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc}' \sin(\alpha + \tilde{\Delta}) - \tilde{\mathcal{P}}_{tc}\tilde{\mathcal{P}}_{uc}' \sin(\alpha - \tilde{\Delta}'), \\ Z_R' &= \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{uc}' \cos(2\alpha + \tilde{\Delta} - \tilde{\Delta}') + \tilde{\mathcal{P}}_{tc}\tilde{\mathcal{P}}_{tc}' \\ &\quad - \tilde{\mathcal{P}}_{uc}\tilde{\mathcal{P}}_{tc}' \cos(\alpha + \tilde{\Delta}) - \tilde{\mathcal{P}}_{tc}\tilde{\mathcal{P}}_{uc}' \cos(\alpha - \tilde{\Delta}'). \end{aligned} \quad (14)$$

As before, we have six independent observables as a function of seven theoretical parameters, so we cannot obtain α . However, one can manipulate Eqs. (12) and (14) to obtain

$$D' = C' \tan 2\alpha + \frac{\tilde{\mathcal{P}}_{tc}'}{\tilde{\mathcal{P}}_{tc}} \frac{B}{2 \cos 2\alpha} - \frac{\tilde{\mathcal{P}}_{tc}}{\tilde{\mathcal{P}}_{tc}'} \frac{B'}{2 \cos 2\alpha}, \quad (15)$$

where

$$\begin{aligned} B &\equiv \frac{1}{2} (-X - Y + X' - Y'), \\ C' &\equiv \frac{1}{2} (-Z_I + Z_I'), \\ B' &\equiv \frac{1}{2} (X - Y - X' - Y'), \\ D' &\equiv \frac{1}{2} (Z_R - Z_R'). \end{aligned} \quad (16)$$

As in Eq. (9) above, the ratio $\tilde{\mathcal{P}}_{tc}/\tilde{\mathcal{P}}_{tc}'$ can be obtained from the ratio of branching ratios for $B_s^0 \rightarrow K^0 \bar{K}^*$ and $B_s^0 \rightarrow \bar{K}^0 K^*$. Thus, Eq. (15) can be used to obtain α , again up to discrete ambiguities.

From the above analysis, we therefore see that the CP phase α can be cleanly extracted from measurements of the decays of B_d^0 and B_s^0 mesons to two different final states consisting of one neutral kaon (i.e., K^0 or any of its excited states) and one neutral anti-kaon (i.e., \bar{K}^0 or any excited state). However, note that the $K^*\bar{K}^*$ final state actually consists of three helicity states. Any of these can be considered a distinct final state for the purposes of our analysis. Thus, by applying our method to two different $K^*\bar{K}^*$ helicity states, α can be obtained from $B_{d,s}^0 \rightarrow K^*\bar{K}^*$ decays alone.

Of course, since they are pure $b \rightarrow d$ penguin decays, the branching ratios for $B_{d,s}^0(t) \rightarrow K^{(*)}\bar{K}^{(*)}$ are expected to be quite small, of order 10^{-6} . Even so, since hadron colliders produce an enormous number of B mesons, such decays should be measurable. Furthermore, in all cases, the kaon or anti-kaon can be detected using its decays to charged π 's or K 's only; this method does not require the detection of π^0 's. Therefore hadron colliders will be able to use this technique to measure α —all that is required is good π/K separation. And if π^0 's can be detected, this simply increases the detection efficiency for the various final states.

Now, the key ingredient in the above method is the use of B_s^0 decays to obtain information about the hadronic parameters of B_d^0 decays. In Eq. (9), we have assumed the equality of a double ratio of matrix elements:

$$\frac{r_t}{r_t^*} \equiv \frac{\langle K^0\bar{K}^0 | H_d | B_d^0 \rangle / \langle K^0\bar{K}^0 | H_s | B_s^0 \rangle}{\langle K^*\bar{K}^* | H_d | B_d^0 \rangle / \langle K^*\bar{K}^* | H_s | B_s^0 \rangle} = 1, \quad (17)$$

where we have defined $H_d \equiv (P_t - P_c)$ and $H_s \equiv (P_t^{(s)} - P_c^{(s)})$. What is the error in making this assumption? Consider first the ratio in the numerator, r_t . The two decays in r_t are related by U -spin, and so r_t is equal to unity in the chiral symmetry limit. Similar observations hold for the ratio in the denominator, r_t^* . We can therefore write

$$r_t = \frac{\langle K^0\bar{K}^0 | H_d | B_d^0 \rangle}{\langle K^0\bar{K}^0 | H_s | B_s^0 \rangle} = 1 + C_{SU(3)},$$

$$r_t^* = \frac{\langle K^*\bar{K}^* | H_d | B_d^0 \rangle}{\langle K^*\bar{K}^* | H_s | B_s^0 \rangle} = 1 + C_{SU(3)}^*,$$

where $C_{SU(3)}$ and $C_{SU(3)}^*$ parametrize the size of $SU(3)$ breaking in these ratios. Thus, we have

$$\frac{r_t}{r_t^*} = 1 + (C_{SU(3)} - C_{SU(3)}^*). \quad (18)$$

Since there is no symmetry limit in which $(C_{SU(3)} - C_{SU(3)}^*) \rightarrow 0$, a priori one would expect this quantity to be of canonical $SU(3)$ -breaking size, i.e., $O(25\%)$. However, as we argue below, there are a number of reasons to expect significant cancellations between $C_{SU(3)}$ and $C_{SU(3)}^*$.

We begin by examining the origin of $SU(3)$ breaking in r_t alone. First, consider the quark-level process underlying the $B_{d,s}^0 \rightarrow K^0\bar{K}^0$ decays, $\bar{b} \rightarrow \bar{d}s\bar{s}$ or $\bar{b} \rightarrow \bar{s}d\bar{d}$. The dominant configuration is the one in which all three final-state quarks are energetic. Thus, in the limit $m_b \rightarrow \infty$ we can neglect the masses of the light quarks, which implies that, at the quark-level, $SU(3)$ breaking is negligible in the decays $B_s^0 \rightarrow K^0\bar{K}^0$ and $B_d^0 \rightarrow K^0\bar{K}^0$. The configuration in which one of the final-state quarks is soft is suppressed by at least $1/E_K$ from the kaon wavefunction, where $E_K = M_B/2$ is the energy of the final-state K^0 or \bar{K}^0 . In addition, the annihilation contributions are suppressed by $1/M_B$. Thus, the subdominant configurations are suppressed by a factor of $1/M_B$ compared to the dominant one. Hence up to corrections of $O((M_{B_d} - M_{B_s})/M_B) \sim 2\%$, the hamiltonians H_d and H_s are the same.

We can therefore write

$$r_t = \frac{\langle K^0\bar{K}^0 | H_d | B_d^0 \rangle}{\langle K^0\bar{K}^0 | H_s | B_s^0 \rangle} = \frac{\langle K^0\bar{K}^0 | H_d | B_d^0 \rangle}{\langle K^0\bar{K}^0 | U^\dagger H_d U | B_s^0 \rangle}, \quad (19)$$

where U is the U -spin operator. Obviously, one would obtain $r_t = 1$ if $SU(3)$ were a good symmetry, since then we would have $U|B_s^0\rangle = |B_d^0\rangle$ and $U|K^0\bar{K}^0\rangle = |K^0\bar{K}^0\rangle$. However, $SU(3)$ is not a good symmetry, and this can affect r_t in 2 distinct ways: (i) “final-state” corrections, $U|K^0\bar{K}^0\rangle \neq |K^0\bar{K}^0\rangle$, and (ii) “initial-state” corrections, $U|B_s^0\rangle \neq |B_d^0\rangle$. In what follows we will examine in turn the size of the $SU(3)$ -breaking effects in each of these areas.

However, before doing so, we note that the sources of $SU(3)$ corrections in r_t^* are very similar to those in r_t : $U|K^*\bar{K}^*\rangle \neq |K^*\bar{K}^*\rangle$ and $U|B_s^0\rangle \neq |B_d^0\rangle$. It is therefore not unreasonable to expect sizeable cancellations between $C_{SU(3)}$ and $C_{SU(3)}^*$, leading to $r_t/r_t^* \simeq 1$.

We first consider $SU(3)$ -breaking effects in the relation $U|K^0\bar{K}^0\rangle = |K^0\bar{K}^0\rangle$. The wavefunction of an energetic K^0 or \bar{K}^0 can be expanded in terms of Fock states as

$$\begin{aligned}\psi_{\bar{K}^0} &= \psi(s\bar{d}) + \psi(s\bar{d}g) + \dots, \\ \psi_{K^0} &= \psi(\bar{s}d) + \psi(\bar{s}dg) + \dots.\end{aligned}\quad (20)$$

In general, the partons inside the energetic kaon are collinear and have small transverse momentum. More precisely, the distribution in the transverse momentum, k_\perp , is peaked at small values of $k_\perp \sim \Lambda_{\text{QCD}}$ [6]. The contributions from higher Fock states, in which the non-valence partons are hard and carry a finite fraction of the kaon momentum, are suppressed by $1/E_K$ because of the additional hard parton propagator in the final state [6]. Hence we assume that the kaon wavefunction is dominated by the valence-quark configuration. In this case, the valence quarks each carry a certain fraction of the total kaon momentum:

$$p_s \approx x p_K, \quad p_d \approx (1-x) p_K, \quad (21)$$

with $0 \leq x \leq 1$.

However, note that, for the calculation of the nonleptonic amplitude, what is relevant is not the full wavefunction of the kaon, $\psi(x, k_\perp)$, but rather its light cone distribution (LCD), $\phi(x, E_K)$, which is related to the wavefunction by $\phi_K(x, \mu) \sim \int^\mu \psi(x, k_\perp) d^2k_\perp$, where $\mu \sim E_K \sim m_b$. Under a U -spin transformation the s and d quarks are interchanged, so that

$$U\psi_K = \psi(d(xp_K)\bar{s}(1-xp_K)). \quad (22)$$

Thus, the U -spin breaking correction from the final state is due to the presence of a piece in the kaon LCD which is antisymmetric under the exchange $x \rightarrow 1-x$.

Now, from QCD we know that the LCDs are symmetric under this exchange as $E_K \rightarrow \infty$ [6]. Therefore, in the $E_K \rightarrow \infty$ limit we have $U|K^0\rangle = |\bar{K}^0\rangle$ and $U|\bar{K}^0\rangle = |K^0\rangle$. To be explicit, the leading-twist kaon LCD $\phi_K(x, \mu)$ can be expanded in terms of Gegenbauer polynomials $C_n^{3/2}$ as follows [7]:

$$\begin{aligned}\phi_K(x, \mu) &= f_K 6x(1-x) \\ &\times \left(1 + \sum_{n=1}^{\infty} a_{2n}^K(\mu) C_{2n}^{3/2}(2x-1) \right),\end{aligned}\quad (23)$$

where the Gegenbauer moments a_n^K are multiplicatively renormalized, change slowly with μ , and vanish as $\mu \rightarrow \infty$. It is the presence of the antisymmetric

piece at scale $\mu \sim m_b$, proportional to odd powers of $(2x-1)$, which will generate $SU(3)$ corrections from the final-state kaons.

Note that, in general, U -spin is *not* a good symmetry for final states in the $E \rightarrow \infty$ limit. For example, it does not hold for $K \leftrightarrow \pi$ transformations because the K and π wavefunctions are still different. Thus, for $K\pi$ final states, one expects to obtain U -spin breaking effects of order f_K/f_π in the $E \rightarrow \infty$ limit. However, U -spin is a good symmetry in the $E \rightarrow \infty$ limit for $K^0 \leftrightarrow \bar{K}^0$ because the K^0 and \bar{K}^0 wavefunctions are the same. Thus, we see that $K^0\bar{K}^0$ is a *special final state* as far as U -spin (i.e., $SU(3)$) is concerned.

We therefore see that $SU(3)$ breaking in the final state is related to the size of the antisymmetric piece of the kaon LCD at the scale of m_b . However, there is indirect experimental evidence that this antisymmetric piece may be absent: the recent measurement of the pion LCD at $\mu^2 \sim 10 \text{ GeV}^2$ [8] shows that the pion LCD is extremely close to its asymptotic form, $\phi_\pi(x) \sim x(1-x)$. (Note: isospin symmetry requires only that the pion LCD be symmetric, not asymptotic.) This suggests that, at the scale $\mu \sim m_b$, the LCDs of the light mesons K and K^* may also be very close to their asymptotic form, i.e., symmetric under the interchange $x \rightarrow 1-x$. If this turns out to be the case, there would be tiny $SU(3)$ breaking from the final states in the ratios r_t and r_t^* , and even tinier $SU(3)$ breaking in the quantity r_t/r_t^* . Note that we only require the antisymmetric parts of the K and K^* LCDs to be absent in order to have tiny $SU(3)$ breaking in r_t/r_t^* . (In fact, if the K and K^* LCDs were measured to be symmetric, it would indicate that the difference between the s -quark and d -quark masses is irrelevant for the K and the K^* wavefunctions.)

The main point here is that this can be tested experimentally: as was done for the pion LCD, one can measure the LCDs of the K and K^* mesons. If they turn out to be symmetric, then, as was argued above, the $SU(3)$ -breaking contribution to r_t and r_t^* will be negligible. On the other hand, if an antisymmetric piece is found, then one needs to estimate its effect on the $B_{d,s}^0 \rightarrow K^{(*)}\bar{K}^{(*)}$ amplitudes. This requires a model calculation, and we will come back to this possibility below.

We now turn to the initial-state corrections, due to $U|B_s^0\rangle \neq |B_d^0\rangle$. In order to treat these quantitatively, we need a framework in which to perform computa-

tions. QCD factorization [9] is arguably the most developed calculational tool, so this is what we shall use. Within QCD factorization, we write the ratio r_t schematically as

$$r_t = \frac{A_{\text{fac}}^d [1 + x^d]}{A_{\text{fac}}^s [1 + x^s]}, \quad (24)$$

where A_{fac}^d and A_{fac}^s are the factorizable contributions to $B_d^0 \rightarrow K^0 \bar{K}^0$ and $B_s^0 \rightarrow K^0 \bar{K}^0$, respectively, and x^d and x^s are the corresponding nonfactorizable contributions. Our definition of A_{fac}^d and A_{fac}^s includes the α_s corrections to naive factorization.

We first consider the nonfactorizable contributions. An example of such effects is the corrections due to hard gluon exchange between the spectator quark and the energetic quarks of the emitted meson. These have been calculated for $B \rightarrow \pi K$ decays [9], and it has been found that the $SU(3)$ -breaking effects are quite small: their size is given by

$$\left(\frac{f_{B_s}}{\lambda_{B_s}} - \frac{f_{B_d}}{\lambda_{B_d}} \right) X, \quad (25)$$

where $M_{B_q}/\lambda_{B_q} = \int \phi_{B_q}(z)/z$, with $\phi_{B_q}(z)$ being the B_q^0 LCD, $q = d, s$. The quantity X depends on the final state. It is straightforward to adapt the calculation of Ref. [9] to $K \bar{K}$ final states: we find that $(f_{B_d}/\lambda_{B_d})X \lesssim 10\%$ (for πK final states, this quantity is $\lesssim 5\%$). Now, in a simple model one can write $f_{B_q} = \mu_q^{3/2}/M_{B_q}^{1/2}$ and $\lambda_{B_q} \sim \mu_q$, where μ_q is the reduced mass, which is different for the B_s^0 and the B_d^0 mesons. Thus, in the heavy-quark limit we have $f_{B_s}/f_{B_d} = \mu_s^{3/2}/\mu_d^{3/2}$ and $(f_{B_s}/\lambda_{B_s})/(f_{B_d}/\lambda_{B_d}) = \mu_s^{1/2}/\mu_d^{1/2}$. Taking $f_{B_s}/f_{B_d} = 1.15$, we find that the $SU(3)$ -breaking correction of Eq. (25) is less than 1%. Thus, within QCD factorization, the $SU(3)$ corrections due to nonfactorizable contributions are negligible. We will henceforth concentrate only on the factorizable contributions A_{fac}^d and A_{fac}^s .

The factorizable contributions can be written as

$$A_{\text{fac}}^d = f_K F_{B_d \rightarrow K} \int T(x) \phi_{\bar{K}}(x) dx, \\ A_{\text{fac}}^s = f_K F_{B_s \rightarrow \bar{K}} \int T(x) \phi_K(x) dx. \quad (26)$$

In the above, $F_{B_d \rightarrow K}$ and $F_{B_s \rightarrow \bar{K}}$ are form factors, while the integrals represent the hadronization of

quarks into a \bar{K}^0 or K^0 . From the above, we see that there are two possible sources of $SU(3)$ breaking: (i) the difference in the \bar{K}^0 and K^0 hadronization, which is related to the difference the \bar{K}^0 and K^0 LCDs, and (ii) the difference in form factors.

We first consider the $SU(3)$ breaking due to the K^0 and \bar{K}^0 LCDs. Since $\phi_{\bar{K}}(x) = \phi_K(1-x)$, it is clear that $SU(3)$ breaking will only occur to the extent that the kaon LCD contains an antisymmetric piece at the scale $\mu \sim m_b$. As has already been discussed, if the kaon LCD turns out to be symmetric, there are no final-state $SU(3)$ corrections to the amplitudes. This is a model-independent result. However, even if $\phi_K(x)$ is found to contain an antisymmetric piece, within QCD factorization it tends not to contribute very much to the overall amplitude. For example, a 50% asymmetry in the LCD of the kaon would only result in a $\sim 4\%$ $SU(3)$ -breaking correction coming from the hard scattering part, $\int T(x) \phi(x) dx$, for the $K^0 \bar{K}^0$ final state at the scale $\mu = m_b$ [9]. (Note that the inclusion of an antisymmetric piece of the kaon LCD introduces a scale dependence in the amplitude, albeit at the α_s^2 level. Thus, since $SU(3)$ corrections cannot depend on the scale μ , a proper $SU(3)$ -breaking calculation should include the full α_s^2 calculation to the nonleptonic amplitude.)

Furthermore, the final state consists of both a K^0 and \bar{K}^0 . Thus, if there is an antisymmetric piece in the kaon LCD, one would expect some cancellation in the amplitude for $B_d^0 \rightarrow K^0 \bar{K}^0$ between the hard scattering, which involves the \bar{K}^0 , and the form factor, which involves the K^0 . A similar argument holds for $B_s^0 \rightarrow K^0 \bar{K}^0$. In fact, the calculation of the $B_d \rightarrow K^0$ and $B_s \rightarrow \bar{K}^0$ form factors using the same antisymmetric piece in the kaon wavefunction results in about a 6% final-state correction to the form factors [10]. This is partially cancelled by the 4% correction coming from the hard scattering part, resulting in an $SU(3)$ correction of $\sim 2\%$ in r_t . Note: since the approach to nonleptonic decays in Ref. [10] (perturbative QCD) is slightly different than that of Ref. [9] (QCD factorization), one has to be careful about combining their results. However, it is reasonable to expect that the net $SU(3)$ breaking will be only about a few percent, as estimated above. A similar analysis holds for the K^* final state and the ratio r_t^* . In addition, the $SU(3)$ breaking in the K and the K^* LCDs is very similar: model calculations [7] find the antisymmetric pieces

to be equal at the level of $\lesssim 10\%$. This indicates that the small $SU(3)$ breaking from the final-state mesons in r_t and r_t^* will come with the same sign and will partially cancel in the ratio r_t/r_t^* .

We therefore see that the various model calculations lead to the conclusion that the final-state $SU(3)$ breaking in r_t/r_t^* is tiny even in the presence of a sizeable antisymmetric piece in the K and the K^* LCDs. Thus, although we admittedly can give no formal proof of this, it seems quite likely that the $SU(3)$ breaking in the final state is indeed very small.

From the above analysis, it appears that the main contribution to $SU(3)$ breaking in r_t (and r_t/r_t^*) comes from the $B \rightarrow K$ form factors. However, QCD factorization says nothing about how to calculate these quantities. Fortunately, we can use experimental measurements to obtain information about the form factors. The main observation is that $B \rightarrow K$ form factors are related to $D \rightarrow K$ form factors.¹ In particular, we note that in the chiral limit and in the heavy-quark limit [11],

$$\frac{F_{B_d \rightarrow K}/F_{B_s \rightarrow \bar{K}}}{F_{\bar{D} \rightarrow K}/F_{\bar{D}_s \rightarrow \bar{K}}} = 1. \quad (27)$$

We therefore conclude that the deviation of this quantity from unity is at most $O[(M_D - M_{D_s}]/M_D) \sim 5\%$. In other words, the measurement of the ratio of $D \rightarrow K$ form factors at $q^2 = 0$ (for example, in semileptonic D decays) will indirectly give us the ratio of the $B \rightarrow K$ form factors at $q^2 = 0$, up to $O(5\%)$ corrections. (There is a slight subtlety here: $q^2 = 0$ for $D \rightarrow K$ form factors corresponds to a kaon energy $E_K = M_D/2$, whereas $q^2 = 0$ for $B \rightarrow K$ form factors implies a larger value of the kaon energy: $E_K = M_B/2$. Thus, if the measurement of $F_{\bar{D} \rightarrow K}/F_{\bar{D}_s \rightarrow \bar{K}}$ yields a deviation from 1 of $X\%$, $F_{B_d \rightarrow K}/F_{B_s \rightarrow \bar{K}} - 1$ is expected to be *less than* $X\%$.)

In fact, this relation between the $B \rightarrow K$ and $D \rightarrow K$ form factors may allow us to deduce that the initial-state $SU(3)$ -breaking corrections are absent: if it is found experimentally that $F_{\bar{D} \rightarrow K}/F_{\bar{D}_s \rightarrow \bar{K}} \simeq 1$, then this implies that $F_{B_d \rightarrow K}/F_{B_s \rightarrow \bar{K}} \simeq 1$, so that the $SU(3)$ -breaking correction of Eq. (18) is $C_{SU(3)} \simeq 0$. If a similar result is found for the $D \rightarrow K^*$ form

factors, one will conclude that $C_{SU(3)}^* \simeq 0$ as well. It is therefore possible to establish experimentally that the $SU(3)$ corrections in r_t , r_t^* and r_t/r_t^* are small.

Suppose instead that the ratio $F_{\bar{D} \rightarrow K}/F_{\bar{D}_s \rightarrow \bar{K}}$ is found to deviate from unity, and similarly for the $D \rightarrow K^*$ form factors. We can therefore write

$$\begin{aligned} \frac{F_{B_d \rightarrow K}/F_{B_s \rightarrow \bar{K}}}{F_{\bar{D} \rightarrow K}/F_{\bar{D}_s \rightarrow \bar{K}}} &= 1 + a \frac{\Delta M_D}{M_D}, \\ \frac{F_{B_d \rightarrow K^*}/F_{B_s \rightarrow \bar{K}^*}}{F_{\bar{D} \rightarrow K^*}/F_{\bar{D}_s \rightarrow \bar{K}^*}} &= 1 + a^* \frac{\Delta M_D}{M_D}, \end{aligned} \quad (28)$$

where a and a^* are numbers of $O(1)$. That is,

$$\begin{aligned} \frac{F_{B_d \rightarrow K}/F_{B_s \rightarrow \bar{K}}}{F_{B_d \rightarrow K^*}/F_{B_s \rightarrow \bar{K}^*}} &= \frac{F_{\bar{D} \rightarrow K}/F_{\bar{D}_s \rightarrow \bar{K}}}{F_{\bar{D} \rightarrow K^*}/F_{\bar{D}_s \rightarrow \bar{K}^*}} \left[1 + (a - a^*) \frac{\Delta M_D}{M_D} \right]. \end{aligned} \quad (29)$$

In other words, the measurement of the $D \rightarrow K$ and $D \rightarrow K^*$ form factors determines the relevant ratio of $B \rightarrow K$ and $B \rightarrow K^*$ form factors up to corrections of $O(5\%)$. Thus, this gives us a method of experimentally measuring the B form factors.

Furthermore, because the various B decays are so similar, one might expect that $a \simeq a^*$ in the above relation, so that the correction to the ratio of ratios of B form factors is in fact smaller than 5%. This is indeed the case: for the pseudoscalar-vector final state, model calculations give $(a - a^*)(\Delta M_D/M_D) < 1\%$ [12].

We note in passing that there are other experimental measurements which probe the size of initial-state $SU(3)$ breaking. For example, neglecting the OZI-suppressed penguin contribution, one expects

$$\frac{\Gamma(B_s^0 \rightarrow \Psi K_S)}{\Gamma(B_d^0 \rightarrow \Psi K_S)} = \left| \frac{V_{cd}}{V_{cs}} \right|^2 (1 + SU(3) \text{ breaking}). \quad (30)$$

Thus, if these two rates are measured to be equal, up to the ratio of CKM factors, this will support the conjecture that initial-state $SU(3)$ -breaking effects in $B \rightarrow K$ transitions are in fact rather small.

Finally, we have information about the $B \rightarrow K$ and $B \rightarrow K^*$ form factors in the limit of $m_b \rightarrow \infty$ and $E_K \rightarrow \infty$. The authors of Ref. [13] showed that in this limit only three form factors, ξ , ξ_{\parallel} and ξ_{\perp} , are necessary to describe $B \rightarrow K$ and $B \rightarrow K^*$ semileptonic transitions. They went on to calculate

¹ We thank C. Bauer, D. Pirjol and I. Stewart for pointing this out to us.

the form factors using QCD sum rules. This approach reproduces the symmetry relations among form factors in the $m_b, E_K \rightarrow \infty$ limit. The three form factors are given by

$$\begin{aligned}\xi &= \frac{1}{f_B} \frac{1}{2E^2} \left[-f_P \phi'(1) I_2(\omega_0, \mu_0) \right. \\ &\quad \left. + f_P \frac{m_P^2}{m_s + m_d} \phi_P(1) I_1(\omega_0, \mu_0) \right], \\ \xi_{\parallel} &= \frac{1}{f_B} \frac{1}{2E^2} \left[-f_V \phi'_{\parallel}(1) I_2(\omega_0, \mu_0) \right. \\ &\quad \left. + f_V^{\perp} m_V h'_{\parallel}(1) I_1(\omega_0, \mu_0) \right], \\ \xi_{\perp} &= \frac{1}{f_B} \frac{1}{2E^2} \left[-f_V^{\perp} \phi'_{\perp}(1) I_2(\omega_0, \mu_0) \right. \\ &\quad \left. + f_V m_V g_{\perp}^{\perp}(1) I_1(\omega_0, \mu_0) \right],\end{aligned}\quad (31)$$

where f_B, f_P, f_V and f_V^{\perp} are decay constants, and $\phi, \phi_P, \phi_{\parallel}, \phi_{\perp}, h'_{\parallel}$ and g_{\perp}^{\perp} are the asymptotic twist-2 and twist-3 LCDs of the K and the K^* . All the initial-state effects are contained in f_B and the integrals $I_{1,2}(\omega_0, \mu_0)$, which are given by

$$I_i(\omega_0, \mu_0) = \exp\left[\frac{2\Lambda}{\mu_0}\right] \int_0^{\omega_0} d\omega \omega^i \exp\left[-\frac{2\omega}{\mu_0}\right], \quad (32)$$

where $\Lambda_q = M_{B_q} - m_b$. Note that Λ_d and Λ_s are in general different and will generate initial-state $SU(3)$ breaking. The parameters ω_0 and μ_0 of the model are defined in Refs. [7,13] and can be taken to be the same for B_d^0 and B_s^0 decays [7], where a tiny flavor dependence for ω_0 has been neglected. It is then straightforward to see that Eq. (31) implies that $(F_{B_d \rightarrow K} / F_{B_s \rightarrow \bar{K}}) / (F_{B_d \rightarrow K^*} / F_{B_s \rightarrow \bar{K}^*}) = 1$, i.e., that all initial-state effects cancel. There are, in principle, $SU(3)$ -breaking corrections to the form factors due to corrections of $O(\alpha_s)$. However, these corrections are themselves very small at $E_K = M_B/2$ [14], and consequently the $SU(3)$ breaking from them is totally negligible. Thus, within the QCD sum rule approach, if the LCDs of the K and the K^* are symmetric at the m_b scale, one has $(F_{B_d \rightarrow K} / F_{B_s \rightarrow \bar{K}}) / (F_{B_d \rightarrow K^*} / F_{B_s \rightarrow \bar{K}^*}) = 1$. Then, using QCD factorization one deduces that $r_t/r_t^* = 1$, up to corrections of $O[(M_{B_d} - M_{B_s})/M_B]$.

To summarize the above discussion: our method assumes the equality of r_t/r_t^* , a double ratio of $B_{d,s} \rightarrow K^{(*)} \bar{K}^{(*)}$ matrix elements. This ratio can deviate from

unity due to flavor $SU(3)$ -breaking effects. Some of these effects (e.g., corrections to the hamiltonian, annihilation contributions, etc.) can be shown to be suppressed by $1/M_B$, and so are expected to be at most $O(\Delta M_B/M_B) \simeq 2\%$. The potentially large corrections are due to final-state effects ($U|K^0 \bar{K}^0\rangle \neq |K^0 \bar{K}^0\rangle$) and initial-state effects ($U|B_s^0\rangle \neq |B_d^0\rangle$). Although we cannot formally prove that these effects are small, all model calculations suggest this to be the case. Furthermore, there are a variety of experimental measurements which can test this conclusion. Taking all the model calculations into account, our best guess is that $SU(3)$ -breaking effects cause r_t/r_t^* to deviate from unity by at most 5%, and it would not be at all surprising if this deviation turns out to be even smaller, say $\lesssim 1\%$.

There is one other source of theoretical uncertainty: in Eq. (8), we have neglected the $(P_u^{(s)} - P_c^{(s)}) V_{ub}^* V_{us}$ term compared to $(P_t^{(s)} - P_c^{(s)}) V_{tb}^* V_{ts}$. The justification is principally the size of the CKM matrix elements: we have

$$|(V_{ub}^* V_{us}) / (V_{tb}^* V_{ts})| \simeq |V_{us}| |V_{ub}| / |V_{cb}| \simeq 0.02,$$

where we have taken $|V_{ub}| / |V_{cb}| = 0.09$ [15]. However, there is also a suppression from the penguin matrix elements: for B_d^0 decays, $|P_u|$ and $|P_c|$ are expected to be at most 50% of $|P_t|$, and are probably smaller. (For example, in Ref. [16] it is found that, for $B_d^0 \rightarrow K^* \bar{K}^*$, $0.14 \leq |P_c - P_u| / |P_t - P_u| \leq 0.54$.) As argued above, this will not change significantly for B_s^0 decays. We therefore conclude that the error made in neglecting the $(P_u^{(s)} - P_c^{(s)}) V_{ub}^* V_{us}$ term in Eq. (8) is less than 1%.

We now turn to an examination of the discrete ambiguities inherent in this method. Consider the pair of decays $B_d^0 \rightarrow K^0 \bar{K}^0$ and $B_d^0 \rightarrow K^* \bar{K}^*$. Let us assume that the true values of the theoretical parameters are

$$\begin{aligned}\mathcal{P}_{tc} &= 1.1, & \mathcal{P}_{uc} &= 0.4, \\ \mathcal{P}'_{tc} &= 1.3, & \mathcal{P}'_{uc} &= 0.2, \\ \Delta &= 40^\circ, & \Delta' &= 70^\circ, & \alpha &= 110^\circ.\end{aligned}\quad (33)$$

Given these inputs, we can calculate the values of the experimental quantities in Eqs. (3) and (4), as well as their primed counterparts. Then, assuming that $\mathcal{P}_{tc}^2 / \mathcal{P}_{tc}'^2$ has been obtained from the decays $B_s^0 \rightarrow$

Table 1

Solutions for α (from Eq. (7)) and hadronic quantities, from measurements of $B_d^0 \rightarrow K^0 \bar{K}^0$ and $B_d^0 \rightarrow K^* \bar{K}^*$, assuming the input values given in Eq. (33)

α	\mathcal{P}_{tc}	\mathcal{P}_{uc}	Δ	\mathcal{P}'_{tc}	\mathcal{P}'_{uc}	Δ'
110°	1.1	0.4	40°	1.3	0.2	70°
160°	3.0	3.5	175.8°	3.6	3.9	177.2°
34.9°	1.7	0.7	25°	2.0	2.3	5°
55.1°	1.2	1.5	10.9°	1.4	0.2	62.2°
101.8°	0.2	1.2	76.6°	0.3	1.4	139.7°
168.2°	1.1	1.8	139.2°	1.3	0.9	107.6°
131.9°	1.0	0.5	47.8°	1.1	1.8	171.2°
138.1°	1.1	1.8	168.1°	1.3	0.3	76°

$K^0 \bar{K}^0$ and $B_s^0 \rightarrow K^* \bar{K}^*$ as in Eq. (9), we can use Eq. (7) to obtain α .

The results are shown in Table 1. There are a total of 16 solutions for α : in addition to the 8 solutions shown in the Table, solutions with $\alpha \rightarrow \alpha + \pi$ are also allowed if one simultaneously takes $\Delta \rightarrow \Delta + \pi$ and $\Delta' \rightarrow \Delta' + \pi$ as well. This large number of discretely ambiguous solutions for α is potentially a serious drawback of this method. However, there are two ways of reducing the discrete ambiguity.

First, one can also consider a different pair of $K^{(*)} \bar{K}^{(*)}$ final states. In this case one expects that the hadronic quantities will take very different values. Because of this, although one still expects a large number of possible solutions for α , these solutions will, in general, be different from those found in Table 1.

This is indeed what happens. For example, consider now the pair of decays $B_d^0 \rightarrow K^0 \bar{K}^*$ and $B_d^0 \rightarrow \bar{K}^0 K^*$, and assume that the hadronic input values are

$$\begin{aligned} \tilde{\mathcal{P}}_{tc} &= 1.2, & \tilde{\mathcal{P}}_{uc} &= 0.2, \\ \tilde{\mathcal{P}}'_{tc} &= 1.0, & \tilde{\mathcal{P}}'_{uc} &= 0.3, \\ \tilde{\Delta} &= 80^\circ, & \tilde{\Delta}' &= 120^\circ. \end{aligned} \quad (34)$$

(Of course, α is assumed to take the same value as in Eq. (33), 110°.) As before, we use these input quantities calculate the values of the observables, and we then solve Eq. (15) to obtain α .

The results are shown in Table 2. As before, we show only 8 solutions for α ; there are another 8 solutions with $\alpha \rightarrow \alpha + \pi$. However, a comparison of Tables 1 and 2 reveals that only two of the eight solutions are common to both sets of processes: 110° (the true solution) and 160°. Thus, by applying the

Table 2

Solutions for α (from Eq. (15)) and hadronic quantities, from measurements of $B_d^0 \rightarrow K^0 \bar{K}^*$ and $B_d^0 \rightarrow \bar{K}^0 K^*$, assuming the input values given in Eq. (34)

α	$\tilde{\mathcal{P}}_{tc}$	$\tilde{\mathcal{P}}_{uc}$	$\tilde{\Delta}$	$\tilde{\mathcal{P}}'_{tc}$	$\tilde{\mathcal{P}}'_{uc}$	$\tilde{\Delta}'$
110°	1.2	0.2	80°	1.0	0.3	120°
160°	3.3	3.6	176.8°	2.8	2.8	174.6°
42°	0.5	1.5	25.9°	0.4	0.9	95.3°
48°	0.5	0.9	135.8°	0.4	1.1	53.4°
15.8°	4.1	4.4	2.6°	3.4	3.5	4.3°
74.2°	1.2	0.3	126.3°	1.0	0.3	79.9°
132.5°	0.5	0.9	42.4°	0.4	1.1	133.9°
137.5°	0.5	1.5	156.4°	0.5	0.8	78.3°

method to several sets of final states, one can reduce the ambiguity in α to a fourfold one.

The second way to reduce the discrete ambiguity is to use the fact that, as discussed above, we expect each of $\mathcal{P}_{uc}/\mathcal{P}_{tc}$, $\mathcal{P}'_{uc}/\mathcal{P}'_{tc}$, $\tilde{\mathcal{P}}_{uc}/\tilde{\mathcal{P}}_{tc}$ and $\tilde{\mathcal{P}}'_{uc}/\tilde{\mathcal{P}}'_{tc}$ to be less than about 0.5 in the SM. This constraint eliminates most of the solutions in Tables 1 and 2. In fact, by combining both methods, one can measure α with only a twofold ambiguity: $\{\alpha, \alpha + \pi\}$. Unless one has knowledge about the strong phases, this discrete ambiguity cannot be further reduced.

Finally, as we have argued above, this method for measuring α includes a theoretical uncertainty of at most 5%. How does this error quantitatively affect the extraction of α ? One can compute this by allowing $\mathcal{P}_{tc}/\mathcal{P}'_{tc}$ and $\tilde{\mathcal{P}}_{tc}/\tilde{\mathcal{P}}'_{tc}$ to vary by $\pm 5\%$ in Eqs. (7) and (15). For the particular cases considered above (Eqs. (33) and (34)), we find that the theoretical uncertainty leads to an error on α of $\pm 12^\circ$. On the other hand, if the theoretical error can be shown to be smaller, say 1%, then the error on α is reduced considerably to $\pm 2^\circ$. Furthermore, for other choices of input parameters, the error on α can be even smaller. This occurs when the hadronic quantities describing the two final states are very different. Thus, the method is most accurate when two very dissimilar final $K^{(*)} \bar{K}^{(*)}$ states are used.

In summary, we have presented a new method for measuring α . It involves the measurements of B_d^0 and B_s^0 decays to $K^{(*)} \bar{K}^{(*)}$ final states. The method is very clean: based on a variety of model calculations, we estimate that the theoretical uncertainty is at most 5%, and it would not be surprising if it turned out to be even smaller. Furthermore, there are several

experimental measurements which can be used to probe the size of the theoretical error. Although there are multiple discrete ambiguities in the extraction of α , by applying the method to several different final states, it is possible to obtain α with a fourfold ambiguity. If an additional (justified) assumption is made, the ambiguity can be reduced to twofold: $\{\alpha, \alpha + \pi\}$. Since this method does not require π^0 detection, it is appropriate for use at hadron colliders.

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References

- [1] B. Aubert et al., BaBar Collaboration, Phys. Rev. Lett. 87 (2001) 091801;
- [2] K. Abe et al., BELLE Collaboration, Phys. Rev. Lett. 87 (2001) 091802.
- [3] For a review, see, for example, P.F. Harrison, H.R. Quinn (Eds.), The BaBar Physics Book, SLAC Report 504, October 1998.
- [4] M. Gronau, D. London, Phys. Rev. Lett. 65 (1990) 3381.
- [5] A.E. Snyder, H.R. Quinn, Phys. Rev. D 48 (1993) 2139; H.R. Quinn, J.P. Silva, Phys. Rev. D 62 (2000) 054002.
- [6] The fact that one needs one piece of theoretical input to obtain CP phase information from $b \rightarrow d$ penguin decays was noted in D. London, N. Sinha, R. Sinha, Phys. Rev. D 60 (1999) 074020.
- [7] G.P. Lepage, S.J. Brodsky, Phys. Rev. D 22 (1980) 2157.
- [8] P. Ball, V.M. Braun, Phys. Rev. D 58 (1998) 094016; P. Ball, JHEP 09 (1998) 005; P. Ball, V.M. Braun, Nucl. Phys. B 543 (1999) 201.
- [9] E.M. Aitala et al., E791 Collaboration, Phys. Rev. Lett. 86 (2001) 4768.
- [10] M. Beneke, G. Buchalla, M. Neubert, C.T. Sachrajda, Nucl. Phys. B 606 (2001) 245.
- [11] C.H. Chen, Phys. Lett. B 520 (2001) 33.
- [12] B. Grinstein, Phys. Rev. Lett. 71 (1993) 3067.
- [13] D. Melikhov, B. Stech, Phys. Rev. D 62 (2000) 014006.
- [14] J. Charles et al., Phys. Rev. D 60 (1999) 014001.
- [15] M. Beneke, T. Feldmann, Nucl. Phys. B 592 (2001) 3.
- [16] D.E. Groom et al., Particle Data Group, Eur. Phys. J. C 15 (2000) 1.
- [17] A. Datta, D. London, C.S. Kim, hep-ph/0105017, submitted to Phys. Rev. D (2001).